A Surface Impedance Approach for Modeling Multilayer Conductors in FDTD

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Abstract **Surface impedance boundary conditions (SIBC's) are implemented in the finite-difference time-doamin (FDTD) method to analyze the electromagnetic field around multilayer conductors. The conducting region is replaced by an equivalent surface where SIBC's are applied locally. To incorporate the equivalent surface impedance into the FDTD code, at first the surface impedance is determined in the frequency domain by replacing the multilayer conductor by a cascade of transmission lines and calculating the total impedance matrix of the network. Following this, a wide-band polynomial approximation of the SIBC's leads to an efficient and recursive solution of the convolution integrals in the time domain. As a numerical example, the impedance matrix of a symmetric three layer conductor is derived and the scattering parameters are compared to the analytical solutions.**

I. Introduction

For many years, the finite-difference time-domain (FDTD) method has been a very popular technique for the solution of Maxwell's equation in scattering, wave propagation, antennas and electromagnetic compatibility problems. The modeling of conducting sheets with finite electrical conductivity plays an important role in the prediction of losses in microwave circuits. Furthermore, an accurate characterization of shielding surfaces is of great intrest for electromagnetic compatibility problems. The analysis of the electromagnetic fields inside conductors requires a cell size smaller than the penetration depth of the current. To this end, the grid has to be refined locally resulting in a large number of unknown field values and a reduction of the time-step due to the Courant stability condition. This method turns out to be very inefficient in most cases. In the literature, several partial solutions to this problem have been proposed [1, 2]. These approaches are limited in frequency range and can only model conductors with a thickness larger than the skin depth. An ultra wide band approach for a single layer conductor and its application to lossy transmission lines was discussed in [3, 4, 5].

However, microwave circuits often include composite conductors due to the fabrication process. Hence, transmission lines or membranes in micro electro-mechanical structures (MEMS) consist of multilayer conductors made of a thick metal layer and thin diffusion/adhesion layers. These thin layers affect the propagation of electromagnetic waves, especially the attenuation and the dissipated power, and therefore have to be taken into account when analyzing circuits or other electronic structures. In this paper, composite conductors are replaced by an impedance matrix computed in the frequency domain and incorporated as a surface impedance boundary condition in the FDTD technique. For an efficient time domain solution of the convolution integrals, a polynomial approximation of the impedance is performed. Herein, a symmetric three layer conductor is utilized to apply and validate the proposed method.

II. Two-port model for SIBC's

In this section, a two-port model for an arbitrary multilayer conductor is developed in the frequency domain. An example for such a composite conductor is shown in Fig. 1. For

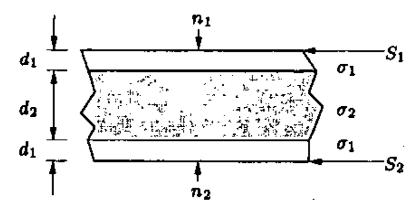

Figure 1: Example for a multilayer conductor

any incident electromagnetic field with an arbitrary incident angle, the field penetration inside the conductor region can be described by the Helmholtz equation. If the conductor is composed by highly conductive materials, the transmission angle in the conductor is perpendicular to the boundary surface. To this end, the penetration of the electric and magnetic field tangential components inside the metal layers is modeled by a plane wave propagation in the direction normal to the conductor surface. For this configuration, the analytical solution is available assuming the composite conductor as a cascade of equivalent transmission lines. The field tangential components at both ends of the equivalent transmission line network are modeled in the frequency domain by the following equations:

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \quad (1)$$

where s is the complex Laplace variable.

Assuming a symmetric multilayer conductor, the impedances

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of the two-port network can easily be determined by assuming a magnetic and electric wall in the center of the conductor for an even and odd excitation, respectively. As a consequence, two impedances, Z^e and Z^o are obtained by a staggered transformation

$$Z_{k+1}^{e,o} = Z_k^{intr} \frac{Z_k^{e,o} + Z_k^{intr}\tanh(\gamma_k d_k)}{Z_k^{intr} + Z_k^{e,o}\tanh(\gamma_k d_k)} \quad (2)$$

of the impedance from the center to the surface boundary of the conductor. In equation (2), $Z_k^{intr} = \sqrt{s\mu/\sigma_k}$ is the intrinsic impedance and $\gamma_k = \sqrt{s\mu\sigma_k}$ is the complex propagation constant of the k^{th} layer bounded by the interfaces k and $k+1$. In view of (2), the impedances of the matrix take the form: $Z_{11} = Z_{22} = \frac{1}{2}(Z^e + Z^o)$ and $Z_{12} = Z_{21} = \frac{1}{2}(Z^e - Z^o)$.

III. FDTD IMPLEMENTATION OF SIBC'S

In this section, the two-port based surface impedance is incorporated in the Yee scheme as shown in Fig. 2. After a polynomial approximation of the impedances in the frequency domain, two possible methods can be applied for an efficient time-domain implementation. The impedance matrix implies that the tangential electric and magnetic field components are located exactly at the boundary surface. In the Yee-scheme however, the electric and magnetic fields are not collocated and for this reason cannot satisfy this requirement. Hence, taking into account the position of the magnetic field, the corrected impedance results in:

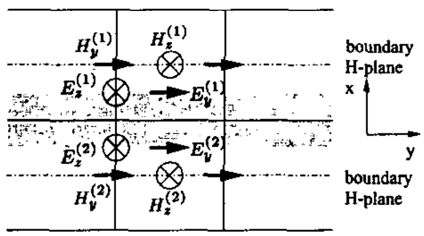

Figure 2: Multilayer conductor in the Yee-mesh

$$Z_{mod}^{e,o} = Z_F \frac{Z^{e,o}}{Z^{e,o}\sinh(j\beta\Delta h) + Z_F\cosh(j\beta\Delta h)} \quad (3)$$

where Z_F is the wave impedance, β the propagation constant and Δh the distance between the two planes containing the electric and magnetic field components. In general, the distance Δh is very small so that the equation (3) can be approximated by linearizing $\sinh(j\beta\Delta h) \approx j\beta\Delta h$ and $\cosh(j\beta\Delta h) \approx 1$. However, for $\omega \ll \left(\sqrt{\sigma_2} Z_F c_0/(\mu d_2)\right)$, a correction of the impedance is not necessary. Typically, equation (3) has to be applied to conductors with a low conductivity and a large total thickness in terms of the cell size.

The impedances in equation (1) contain the transcendental functions $\sqrt{x}\tanh(x)$ and $\sqrt{x}\tanh(x)$ due to the staggered impedance transformation from the center to the surface boundary of the conductor. In order to obtain a rational functional representation of the impedances $Z_{i,j}$, both transcendental functions are replaced by a third order rational approximation [1]

$$\frac{a_0 + a_1 x_1 + a_2 x^2 + a_3 x^3}{b_0 + b_1 x_1 + b_2 x^2 + b_3 x^3} \quad (4)$$

with coefficients a_i and b_i given in Table 1. After replacing

	$\sqrt{x}\coth(x)$			
a_i	0.201833	0.0922242	4.30272e-3	2.99416e-5
b_i	0.201833	0.0249482	4.70416e-4	8.47668e-7
	$\sqrt{x}\tanh(x)$			
a_i	1.19000e-6	0.0478011	5.36751e-3	6.59864e-5
b_i	0.0478138	0.0212851	8.00010e-4	2.32392e-6

Table 1: Coefficients for the third order rational approximation in the interval $x \in [0, 40]$

the transcendental functions by their approximations, the elements of the impedance matrix in equation (1) become rational functions of the Laplace variable s.

To include equation (1) into the FDTD method, two different approaches are pursued: the *recursive convolution method* and the *billinear transformation* into the Z-domain.

Adopting the *recursive convolution method*, the impedances are decomposed into partial fractions in the Laplace domain:

$$\hat{Z}_{ij} = \sum_i \frac{a_i}{s + c_i} + \sum_i \frac{a_i + s b_i}{c_i + s d_i + s^2 e_i} \quad (5)$$

with $\hat{Z} = Z/s$ and a_i, b_i, c_i, d_i, e_i being real coefficients. To this end, the inverse Laplace transform of equation (5) enables a recursive implementation of the convolution integrals. Applying the surface boundary condition to the Yee mesh in the time domain and taking the field components according to Fig. 2, the electric field components can be expressed by the time derivatives of the magnetic field

$$\pm E_{y,z}^{(1)}|^n = -\hat{Z}_{11} * \frac{\partial}{\partial t} H_{z,y}^{(1)}|^n + \hat{Z}_{12} * \frac{\partial}{\partial t} H_{z,y}^{(2)}|^n \quad (6)$$

$$\pm E_{y,z}^{(2)}|^n = -\hat{Z}_{21} * \frac{\partial}{\partial t} H_{z,y}^{(1)}|^n + \hat{Z}_{22} * \frac{\partial}{\partial t} H_{z,y}^{(2)}|^n \quad (7)$$

Together with Faraday equations

$$-\frac{\partial}{\partial t} H_{y,z}^{(1)}|^n \mu = \nabla \times E^{(1)}|_{y,z}^n \quad (8)$$

$$-\frac{\partial}{\partial t} H_{y,z}^{(2)}|^n \mu = \nabla \times E^{(2)}|_{y,z}^n \quad (9)$$

[1] Approximation performed in the symbolic algebra program MAPLE VII using the module *minimax* of the package *numapprox*. Minimizing the relative error in an specified intervall for x was the optimization criterion.

the system of equations (6)-(9) can be solved for the tangential magnetic field components leading to a boundary condition for the magnetic field.
In the second case, the *billinear transformation* into the Z-domain

$$s = \frac{2(1 - z^{-1})}{\Delta t(1 + z^{-1})} \tag{10}$$

is employed in equation (1). After some algebraic manipulations, rational fractions in z for the impedances Z_{ij} are obtained which yield directly a discrete form for the impedance matrix:

$$\pm \sum_{i=0}^{M} a_i E_{y,z}^{(1)}|^{n-i+1} = \sum_{i=0}^{M} b_i \frac{\partial}{\partial t} H_{z,y}^{(1)}|^{n-i+1} + \sum_{i=0}^{M} c_i \frac{\partial}{\partial t} H_{z,y}^{(2)}|^{n-i+1} \tag{11}$$

$$\pm \sum_{i=0}^{M} a_i E_{y,z}^{(2)}|^{n-i+1} = \sum_{i=0}^{M} c_i \frac{\partial}{\partial t} H_{z,y}^{(1)}|^{n-i+1} + \sum_{i=0}^{M} b_i \frac{\partial}{\partial t} H_{z,y}^{(2)}|^{n-i+1} \tag{12}$$

Together with the Faraday equations (8) and (9), the system of equations (11) and (12) can be solved for the magnetic field components. This method has a significant advantage in that a decomposition in partial fractions is not required and only a straightforward algebraic calculation has to be implemented.

IV. Symmetric Three Layer Conductor

As an numerical example, a symmetric conductor consisting of three layers as shown in Fig. 1 is analyzed. The center conductor has a conductivity $\sigma_2 = 2.64$ S/m and a thickness $d_2 = 32$ mm. The conductivity of the thin layer on the top and the bottom is 1.32 S/m and its thickness is 4 mm. For this structure, the elements of the impedance matrix are approximated by rational fractions in the s domain according to section III. The relative error of the magnitude as a function of frequency is shown in Fig. 3 and Fig. 4 for the impedances Z_{11} and Z_{12}, respectively. The frequency in the diagram is normalized to a reference frequency f_g, where the skin depth is half of the conductor thickness d_2. Whereas the impedance Z_{11} can be approximated with an error less than 0.045 % over a wide frequency range, the impedance Z_{12} shows a larger error in a significant smaller frequency range. An error of 0.35 % can only be achieved from DC to 10 times the reference frequency f_g. The reason is that the exponential decay of the impedance Z_{12} leads to an enormous increase of the error in the case of a polynomial approximation.
The rational approximation of the surface impedance is not limited to the thickness and conductivity given in the numerical example. The method can also handle multilayer conductors, where the top and bottom layers are extremly thin compared to the center conductor. However, a different relative error is obtained as a function of the frequency.
Using the recursive convolution method, the scattering parameters of the composite conductor with a total thickness of 40 mm were extracted from the FDTD simulation and compared to the analytic solution of the TEM problem. A cell size of 1 mm is chosen for the grid. The magnitude and the phase of the reflection coefficient is pictured in Fig. 5 and Fig. 6. The simulation results agree with the exact solution and only show a very small discrepancy at higher frequencies. The staircase approximation of the impedances in the time-domain causes an error in the convolution integrals at higher frequencies when the time-step approaches the Nyquist criterion. Furthermore, the accuracy for the algebraic manipulations in the decomposition of the rational function into partial fractions is limited. The higher the degree of the polynomial the less accurate are the coefficients in the partial fractions

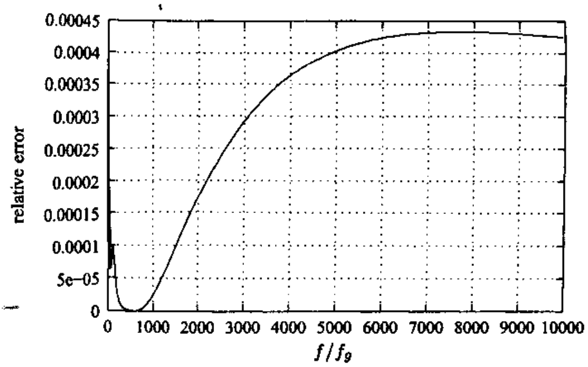

Figure 3: Relative error of $|Z_{11}|$; frequency is normalized to $f_g = 4/(\pi\mu\sigma_2 d_2^2)$.

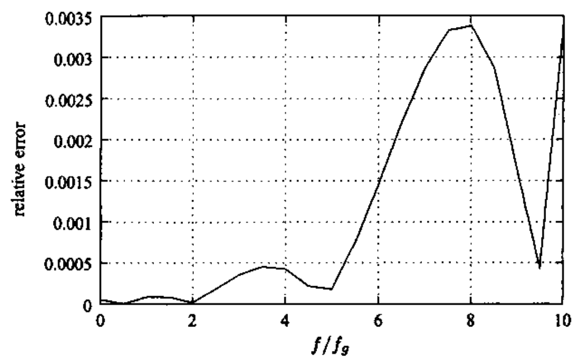

Figure 4: Relative error of $|Z_{12}|$;frequency is normalized to $f_g = 4/(\pi\mu\sigma_2 d_2^2)$.

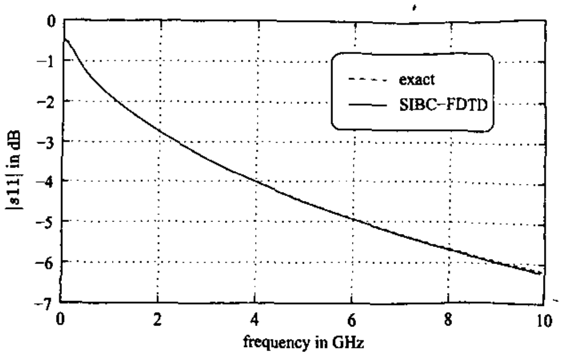

Figure 5: Scattering parameters of a symmetric three layer conductor.

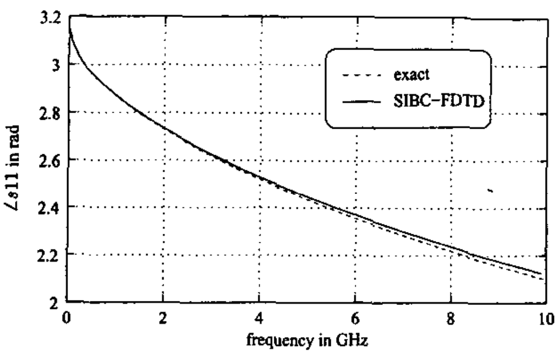

Figure 6: Scattering parameters of a symmetric three layer conductor.

due to the finite number of digits per floating point number. In Fig. 7, the magnitude of the reflection coefficient is shown when the correction of the impedance according to equation (3) is not applied. In this case, a symmetric excitation at both ports is assumed. Without a correction of the impedance, an error of 7.3 % in the magnitude and 2.1 % in the phase results at 10 GHz. Taking into account, that the electric and magnetic fields are on different layers, the error is reduced to 0.36 % and 0.33 % in magnitude and phase, respectively.
The billinear method leads to the same scattering parameters with an error comparable to the convolution method. However, the billinear transformation requires a high precision for the floating point numbers when the filter coefficients are calculated. For higher order polynomials resulting from conductor with several multilayers, this method becomes inefficient. The convolution technique turns out to be a solid and stable method and also can be applied to more than three layers of arbitrary lengths.

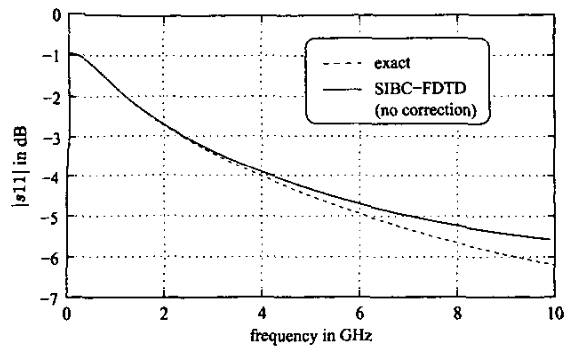

Figure 7: Symmetric three layer conductor: Reflection for a symmetric excitation using no correction term

CONCLUSION

A SIBC for multilayer conductor was introduced in this paper. The SIBC is based on the impedance matrix of a transmission line network. A rational approximation enables a stable and efficient recursive convolution method in FDTD. The proposed method has been tested with a symmetric three layer conductor and the scattering parameters were validated by the analytical solutions. Based on this method, transmission line losses in microwave circuits and electromagnetic compatibility problems can be considered as well.

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REFERENCES

[1] J. Beggs et al., "Finite difference time domain implementation of surface impedance boundary conditions", *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 49-56, Jan. 1992.

[2] L.-K. Wu and L.T. Han, "Implementation and application of resistive sheet boundary condition in the finite difference time domain method", *IEEE Trans. Antennas Propagat.*, vol. 40, pp. 628-633, June 1992.

[3] W. Thiel, "A Surface Impedance Approach for Modeling Transmission Line Losses in FDTD", *IEEE Microwave Guided Wave Lett.*, vol. 10, no. 3, pp. 89-91, March 2000.

[4] A. Lauer and I.Wolff,"A conducting sheet model for efficient wide band FDTD analysis of planar waveguides and circuits", *Microwave Symp. Dig.*, 1999, pp. 1589-1592.

[5] M. Feliziani, F. Maradei and G. Tribellini, "Field analysis of penetrable conductive shields by the Finite-Difference Time-Domain (FDTD) method with Impedance Network Boundary Conditions (INBC's)", *IEEE Trans. Electromagn. Compat.*, vol. 41, no. 4, pp. 307-319, Nov. 1999.